Steps to calculate the respiration rate from the standard DEB expressed in an energy-length-time framework

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1 Step summary

1. we define the elemental composition of the different organic and mineral compounds
2. we write the macrochemical reactions
3. we calculate $\dot{p}_A$, $\dot{p}_D$, $\dot{p}_G$ at each length data
4. we convert the energy fluxes into mass fluxes
5. we calculate the mineral fluxes from the elemental composition of the different organic compounds and the law of mass conservation
6. we calculate the yield coefficients for each transformation (optional)

2 Elemental composition of the mineral and organic compounds

4 mineral (CO_2 noted C, H_2O noted H, O_2 noted O and N-waste noted N) and 4 organic compounds (Food noted X, Structure noted V, Reserve noted E and Faeces noted P) with 4 elements (C, H, O, N)

NOTA : if we want to consider more than 4 elements, say 5 elements, we need 5 mineral compounds to complete the mass balance

NOTA2 : the order used for the mineral (C then H then O then N) and the organic compounds (X then V then E then P) matters as we deal with matrix calculations.

$$\boldsymbol{n}_{\mathcal{M}} = \begin{pmatrix} n_{CC} & n_{CH} & n_{CO} & n_{CN} \\ n_{HC} & n_{HH} & n_{HO} & n_{HN} \\ n_{OC} & n_{OH} & n_{OO} & n_{ON} \\ n_{NC} & n_{NH} & n_{NO} & n_{NN} \end{pmatrix} \quad ; \quad \boldsymbol{n}_{\mathcal{O}} = \begin{pmatrix} n_{CX} & n_{CV} & n_{CE} & n_{CP} \\ n_{HX} & n_{HV} & n_{HE} & n_{HP} \\ n_{OX} & n_{OV} & n_{OE} & n_{OP} \\ n_{NX} & n_{NV} & n_{NE} & n_{NP} \end{pmatrix} \tag{1}$$

As an example, we have:

$$\boldsymbol{n}_{\mathcal{M}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 3 \\ 2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{n}_{\mathcal{O}} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1.8 & 1.8 & 1.8 & 1.8 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix} \tag{2}$$

Question: How does $\boldsymbol{n}_{\mathcal{M}}$ change if N-waste is urea (e.g. mammals) $CO(NH_2)_2$, or uric acid $C_5H_4O_3N_4$ (e.g. birds)?

3 Macrochemical reactions

3.1 Assimilation

$$Y^A_{XE}X + Y^A_{OE}O \to Y^A_{EE}E + Y^A_{PE}P + Y^A_{CE}C + Y^A_{HE}H + Y^A_{NE}N \tag{3}$$

3.2 Dissipation

$$E + Y^D_{OE}O \to Y^D_{CE}C + Y^D_{HE}H + Y^D_{NE}N \tag{4}$$

3.3 Growth

$$E + Y^G_{OE}O \to Y^G_{VE}V + Y^G_{CE}C + Y^G_{HE}H + Y^G_{NE}N \tag{5}$$

4 Calculation of $\dot{p}_A$, $\dot{p}_D$, $\dot{p}_G$

see Kooijman (2010)

5 Calculation of fluxes of organic compounds from energy fluxes

A compound can be substrate and/or product. As a convention, substrate fluxes will be negative (i.e. they disappear)

Food, X : substrate (< 0)

$$\begin{aligned} \dot{J}_X = &\quad \dot{J}_{XA} \quad + \dot{J}_{XD} + \dot{J}_{XG} \\ = &(-\eta_{XA})\dot{p}_A + 0\dot{p}_D + 0\dot{p}_G \end{aligned} \tag{6}$$

with

$$\eta_{XA} = \frac{1}{\mu_X}\frac{1}{\kappa_X} \tag{7}$$

Structure, V : product (> 0)

$$\begin{aligned}\dot{J}_V &= \dot{J}_{VA} + \dot{J}_{VD} + \dot{J}_{VG} \\ &= 0\dot{p}_A + 0\dot{p}_D + \eta_{VG}\dot{p}_G\end{aligned} \tag{8}$$

with

$$\eta_{VG} = \frac{1}{\mu_V}\kappa_G \tag{9}$$

$$= \frac{d_V}{w_V[E_G]} \tag{10}$$

Reserve, E : product (> 0) and substrate (< 0)

$$\begin{aligned}\dot{J}_E &= \dot{J}_{EA} + \dot{J}_{ED} + \dot{J}_{EG} \\ &= \eta_{EA}\dot{p}_A + (-\eta_{ED})\dot{p}_D + (-\eta_{EG})\dot{p}_G\end{aligned} \tag{11}$$

with

$$\eta_{EA} = \frac{1}{\mu_E} \tag{12}$$

$$\eta_{ED} = \frac{1}{\mu_E} \tag{13}$$

$$\eta_{EG} = \frac{1}{\mu_E} \tag{14}$$

Faeces, P : product (> 0)

$$\begin{aligned}\dot{J}_P &= \dot{J}_{PA} + \dot{J}_{PD} + \dot{J}_{PG} \\ &= \eta_{PA}\dot{p}_A + 0\dot{p}_D + 0\dot{p}_G\end{aligned} \tag{15}$$

with

$$\eta_{PA} = \frac{1}{\mu_P}\frac{\kappa_P}{\kappa_X} \tag{16}$$

which in matrix notation can be summarized as

$$\dot{\boldsymbol{J}}_{\mathcal{O}} = \boldsymbol{\eta}_{\mathcal{O}}\dot{\boldsymbol{p}} \tag{17}$$

$$\dot{\boldsymbol{J}}_{\mathcal{O}} = \begin{pmatrix} \dot{J}_X \\ \dot{J}_V \\ \dot{J}_E \\ \dot{J}_P \end{pmatrix} \quad ; \quad \boldsymbol{\eta}_{\mathcal{O}} = \begin{pmatrix} -\eta_{XA} & 0 & 0 \\ 0 & 0 & \eta_{VG} \\ 1/\mu_E & -1/\mu_E & -1/\mu_E \\ \eta_{PA} & 0 & 0 \end{pmatrix} \quad ; \quad \dot{\boldsymbol{p}} = \begin{pmatrix} \dot{p}_A \\ \dot{p}_D \\ \dot{p}_G \end{pmatrix} \tag{18}$$

6 Calculation of mineral fluxes

The law of mass conservation gives

$$\boldsymbol{n}_{\mathcal{M}}\dot{\boldsymbol{J}}_{\mathcal{M}} + \boldsymbol{n}_{\mathcal{O}}\dot{\boldsymbol{J}}_{\mathcal{O}} = \boldsymbol{0} \tag{19}$$

thus, we have

$$\dot{\boldsymbol{J}}_{\mathcal{M}} = -\boldsymbol{n}_{\mathcal{M}}^{-1}\boldsymbol{n}_{\mathcal{O}}\dot{\boldsymbol{J}}_{\mathcal{O}} \tag{20}$$

7 Calculation of yield coefficients

For each transformation, the law of mass conservation applies, e.g.:

$$\boldsymbol{n}_{\mathcal{M}}\boldsymbol{Y}^{G}_{\mathcal{M}E} + \boldsymbol{n}_{\mathcal{O}}\boldsymbol{Y}^{G}_{\mathcal{O}E} = \boldsymbol{0} \tag{21}$$

which is equivalent to

$$\boldsymbol{Y}^{G}_{\mathcal{M}E} = -\boldsymbol{n}_{\mathcal{M}}^{-1}\boldsymbol{n}_{\mathcal{O}}\boldsymbol{Y}^{G}_{\mathcal{O}E} \tag{22}$$

with

$$\boldsymbol{Y}^{G}_{\mathcal{M}E} = \begin{pmatrix} Y^{G}_{CE} = \dot{J}_{CG}/\dot{J}_{EG} \\ Y^{G}_{HE} = \dot{J}_{HG}/\dot{J}_{EG} \\ Y^{G}_{OE} = \dot{J}_{OG}/\dot{J}_{EG} \\ Y^{G}_{NE} = \dot{J}_{NG}/\dot{J}_{EG} \end{pmatrix} \quad \text{and} \quad \boldsymbol{Y}^{G}_{\mathcal{O}E} = \begin{pmatrix} Y^{G}_{XE}=\dot{J}_{XG}/\dot{J}_{EG}= & 0 \\ Y^{G}_{VE}=\dot{J}_{VG}/\dot{J}_{EG}= & -y_{VE} \\ Y^{G}_{EE}=\dot{J}_{EG}/\dot{J}_{EG}= & 1 \\ Y^{G}_{PE}=\dot{J}_{PG}/\dot{J}_{EG}= & 0 \end{pmatrix} \tag{23}$$

we obtain

$$\boldsymbol{Y}^{G}_{\mathcal{M}E} = \begin{pmatrix} -0.2 \\ -0.12 \\ 0.21 \\ -0.04 \end{pmatrix} \tag{24}$$

We can write

$$E + 0.21O \rightarrow 0.8V + 0.2C + 0.12H + 0.04N \tag{25}$$