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Parameter Identifiability

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ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΡΗΤΗΣ

UNIVERSITY OF CRETE

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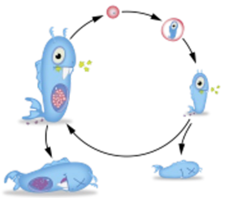
Baton Rouge, Louisiana, United States

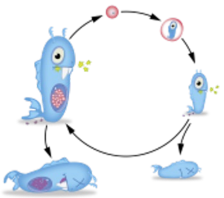
deb2023.sciencesconf.org

Outline



- Structural and practical identifiability
- Handling non- identifiable parameters
- Quantifying accuracy of parameters of deterministic models

Parameter estimation- challenges



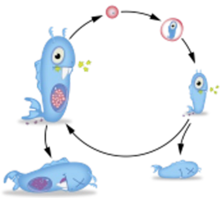
- Choice of appropriate loss functions

  to fit multiple models, which share parameters, to multiple data sets, which may differ in dimensions, in a single parameter estimation.

- Identification of model parameters by the available data

- Quantification of accuracy of the parameter estimates

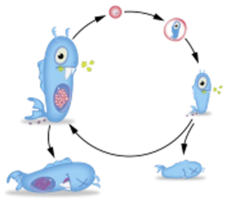
Parameter identification problem



- The inability to identify a best set of estimates.
- Two or more sets of parameter values generate the same observations.
- Parameter estimates may vary by orders of magnitude without significantly influencing the quality of fit.

Sources of the problem

Insufficient or noisy data
Strong parameter correlation



Identifiability analysis

structural identifiability analysis - *a priori*

- **fix** non-identifiable parameters
- substitution of the non-identifiable parameters by appropriately chosen combinations of parameters (**compound parameters**)
- use of **pseudo-data**

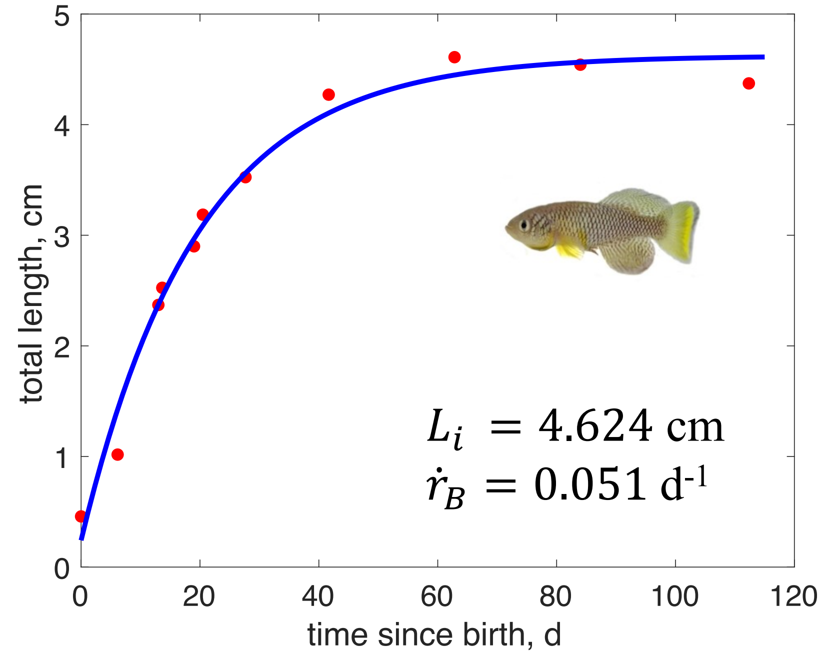
Example

Von Bertalanffy equation

$$L(a) = L_\infty - (L_\infty - L_b)\exp\{-\dot{r}_B a\}$$

$$L_\infty = f\frac{\kappa\{\dot{p}_{Am}\}}{[\dot{p}_M]}, \quad \dot{r}_B = \frac{[\dot{p}_M]}{3([E_G] + f\kappa[E_m])}$$

Estimate compound parameters L_i and $\dot{r}_B$ with L_b=0.24 cm



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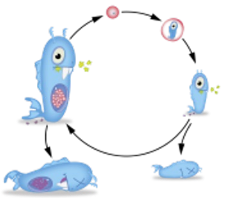
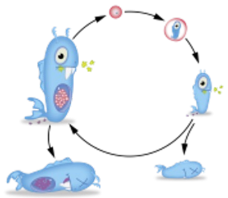


Identifiability analysis

- practical identifiability analysis - *a posteriori*
  non-identifiabilities are detected by fitting the model to data and investigating parameter estimates



Identifiability analysis

Tools

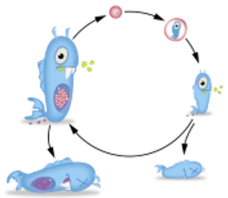
- sensitivity analysis

  SA investigates how the variation in the output of a model can be attributed to variations of its parameters/input (forcing) variables

- confidence intervals

  A confidence interval is an interval (hopefully a small) that contains a certain percentage of the total distribution of that parameter

Local Sensitivity Analysis



LSA assesses which parameters are the most influential around a reference value

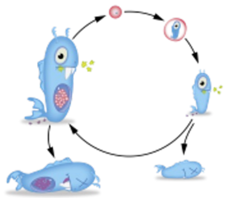
- change the value of a parameter while keeping the others constant

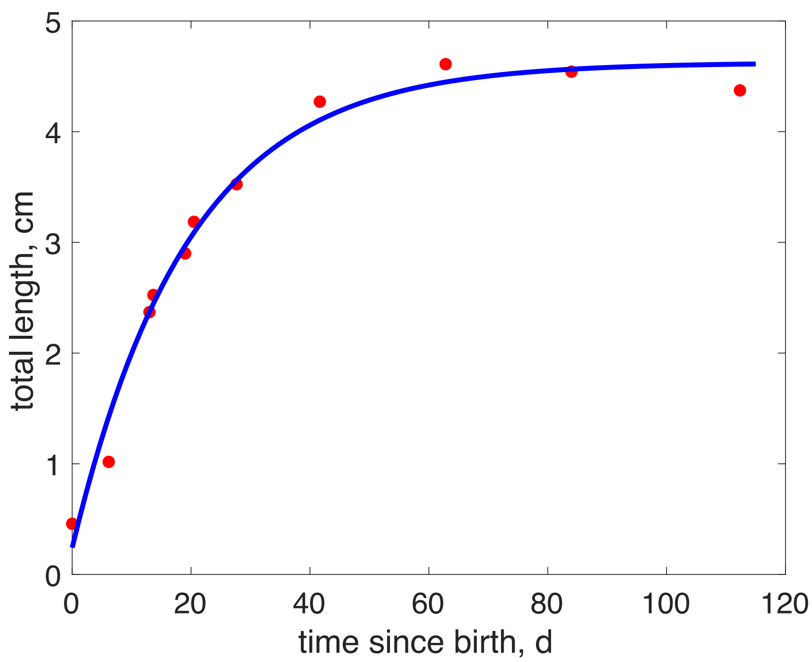
Sensitivity index	Elasticity coefficient
$s_\theta(t) = \dfrac{\partial Y(t)}{\partial \theta}$	$e_\theta(t) = \dfrac{\theta}{Y(t)} \dfrac{\partial Y(t)}{\partial \theta}$

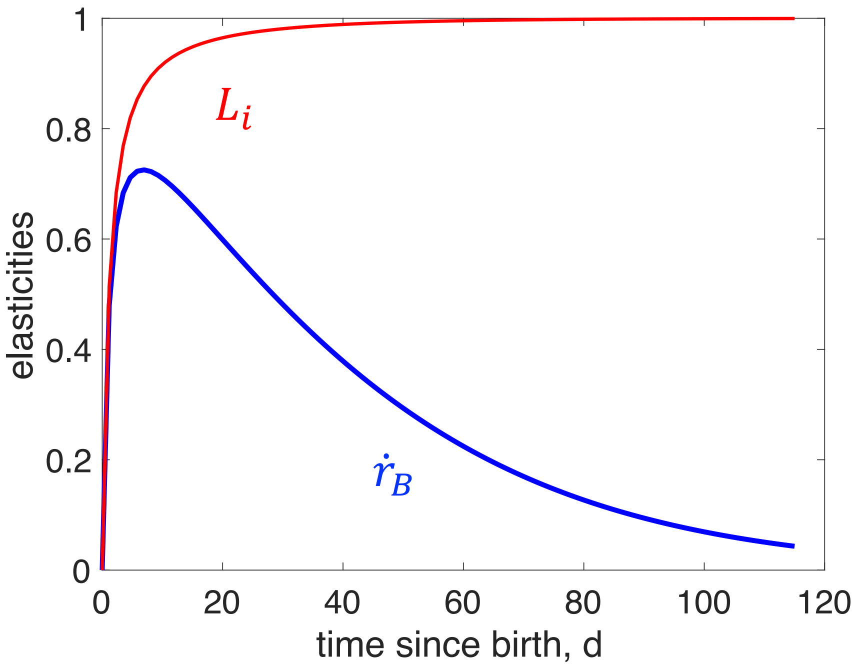
Y: output variable

θ: parameter

VB curve – local sensitivity

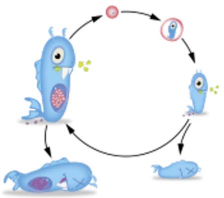






perturbation factor 10^{-3}

Effects of pseudo-data



- Elasticity coefficients

θ a core parameter to be estimated

θ_0 a typical value of the pseudo-data

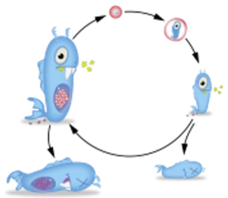
$$e_\theta = \frac{\frac{\hat{\theta}_1 - \hat{\theta}_0}{\hat{\theta}_0}}{\frac{\theta_0(1+\alpha) - \theta_0}{\theta_0}} = \frac{\hat{\theta}_1 - \hat{\theta}_0}{\alpha \hat{\theta}_0}$$

$\hat{\theta}_0$ estimate of θ given the pseudo-data θ_0

$\hat{\theta}_1$ estimate of θ given the pseudo data $\theta_0(1+\alpha)$

α percentage change in pseudo-value

The smaller the (absolute) elasticity, the less the role of that particular pseudo-data in the estimation of the parameter

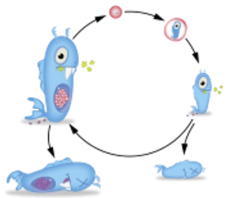


Confidence intervals on the estimated parameters

A confidence interval is an interval that contains a specified percentage of the distribution of that parameter

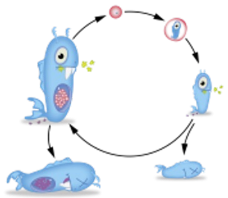
- marginal confidence set
- a profile-based method

The profile method



A 2-step procedure for the assessment of marginal confidence sets

1. Obtain the profile of the loss function for a parameter
2. Obtain the distribution function of the global minimum of the loss function (calibration step)



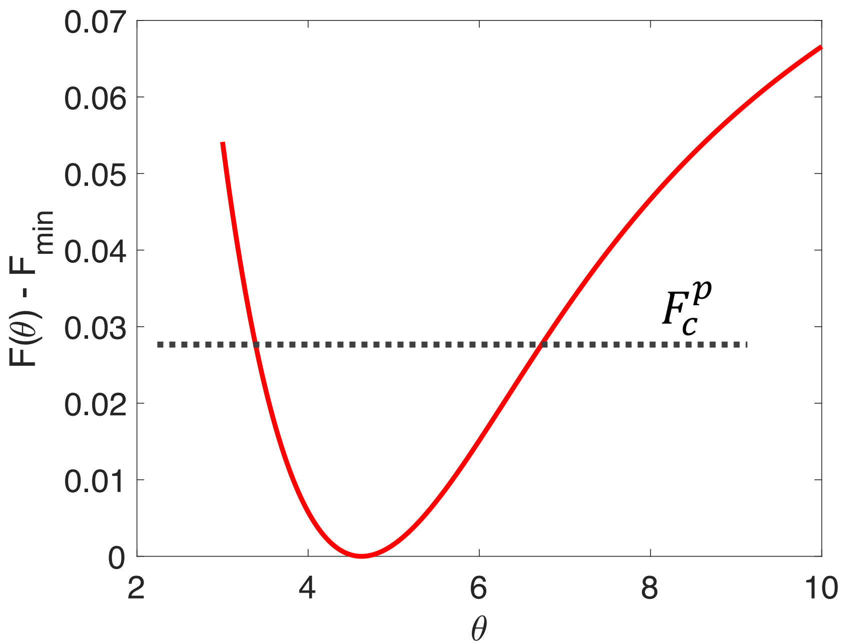
Profile of the loss function for a parameter θ

Start from the point estimate, i.e. the parameter $\hat{\theta}$ that minimizes the loss function $F_{min} = F(\hat{\theta})$, and move the parameter step-wise up or down, while estimating the other parameters using continuation.

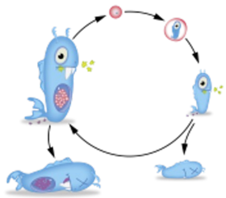
Plot the profile of the loss function $F(\theta)$ as function of the parameter of interest.

A marginal profile-based confidence set for the parameter θ with confidence level p, 0<p<1,is the set of values of θ defined as

$$\{\theta: F(\theta) - F_{min} \leq F_c^p\}$$



Calibration step for the interval estimation



Generate **Monte Carlo data-sets** by (generally) adding centered log-normally distributed scatter to the predictions for each data-point (zero- and uni-variate data), with the same coefficient of variation (cv) as the data shows from these predictions.

log-normal distribution with mean $p_{\rm ij}$ and variance $(cv \cdot p_{\rm ij})^2$

$$cv = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{n_i}\sum_{j=1}^{n_i}\frac{|d_{ij} - p_{ij}|}{p_{ij}}$$

For each Monte Carlo dataset estimate the parameters and compute the (global) minimum, F_{min}, of the loss function

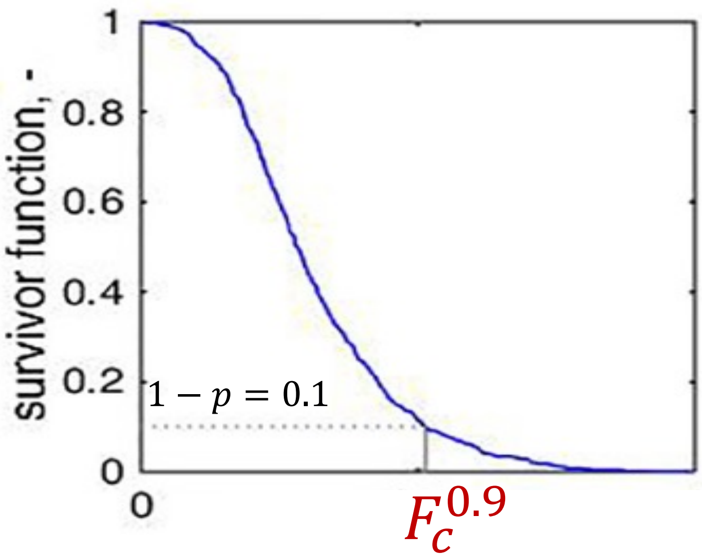
Determine the cumulative distribution of the global minimum of the loss function

The **critical value** F_c^p that corresponds to a certain **confidence level** $\boldsymbol{p}$

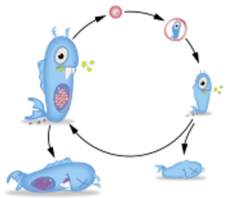
$$P(F \leq F_c^p) = p$$

or

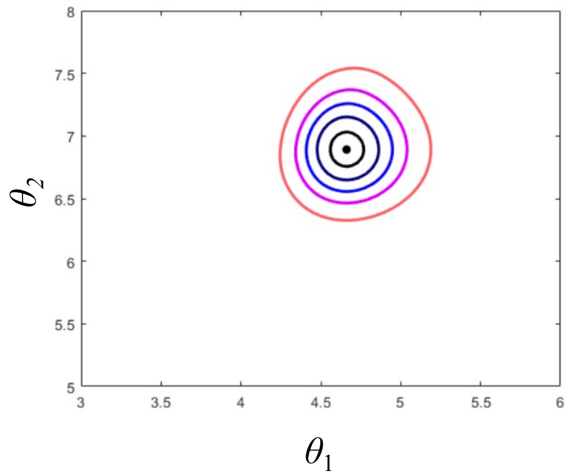
$$S(F_c^p) = P(F > F_c^p) = 1 - p$$

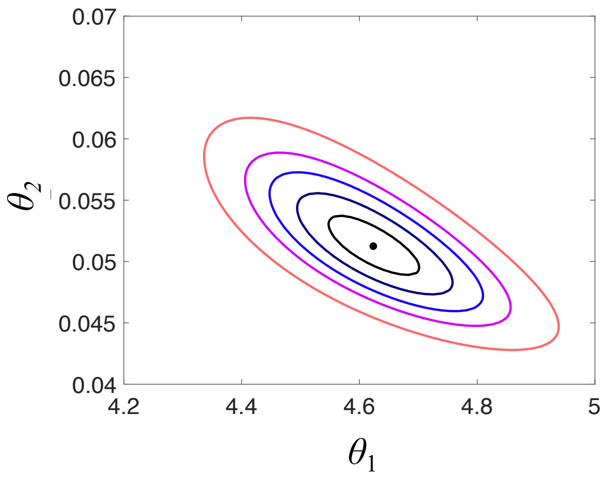


Confidence regions in 2D space



Contours of the loss function at various confidence leveles





Example

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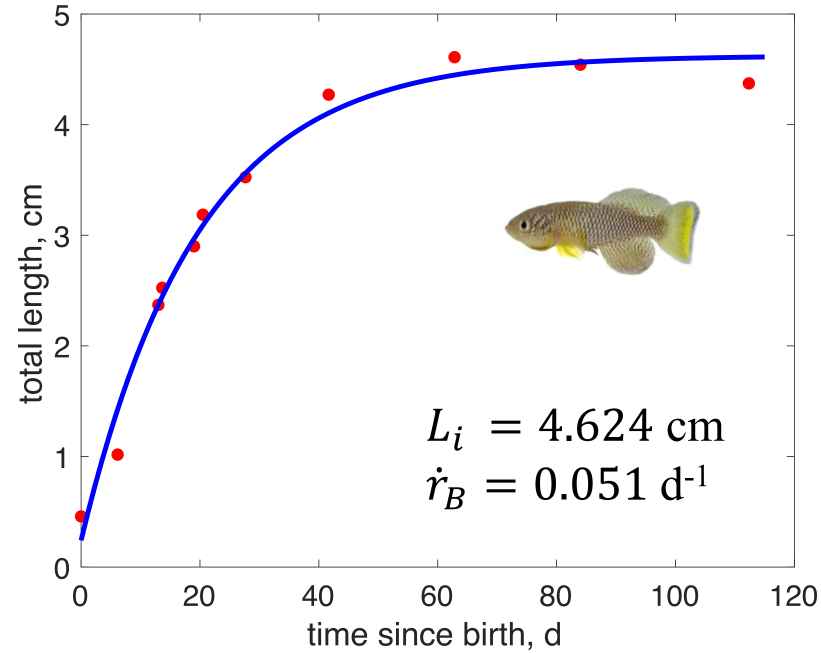
Estimate compound parameters
L_i and $\dot{r}_B$ with L_b=0.24 cm



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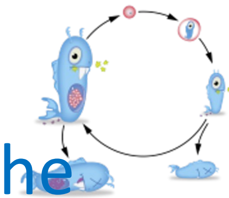
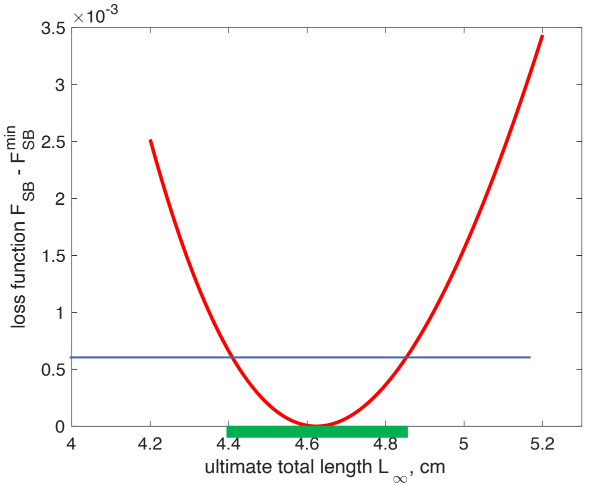
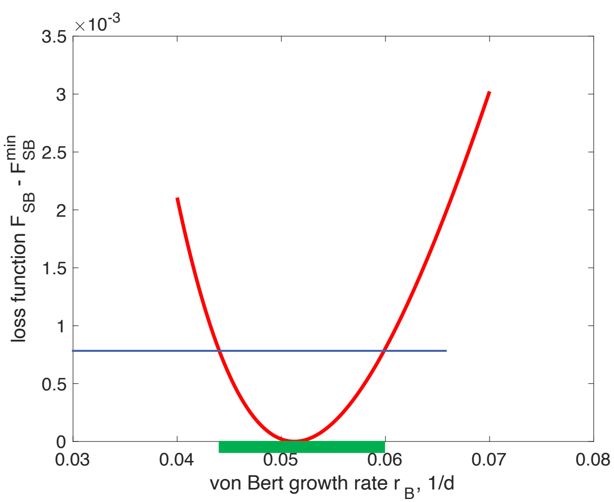
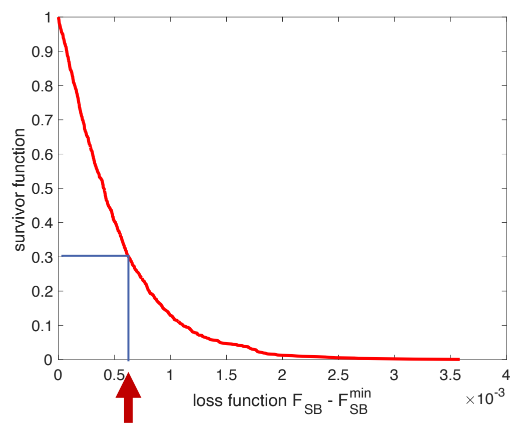


The marginal profiles of the loss functions





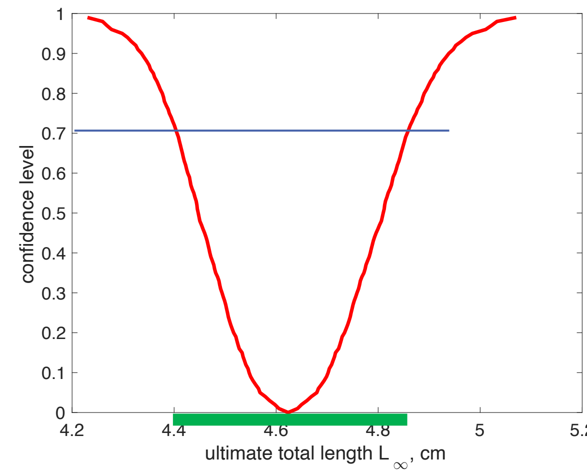
Survivor function of the loss function

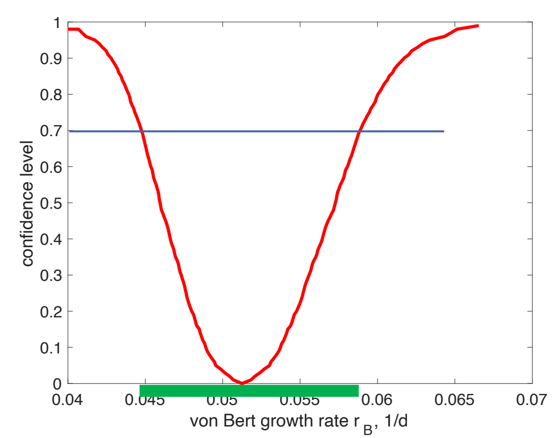


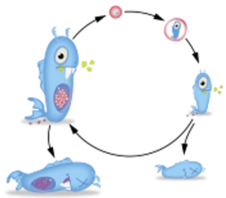
$F_c^{0.7}$

In green 70% CI

Boundaries of the confidence interval







Example – reduced data set

sampled from the initial part of the curve

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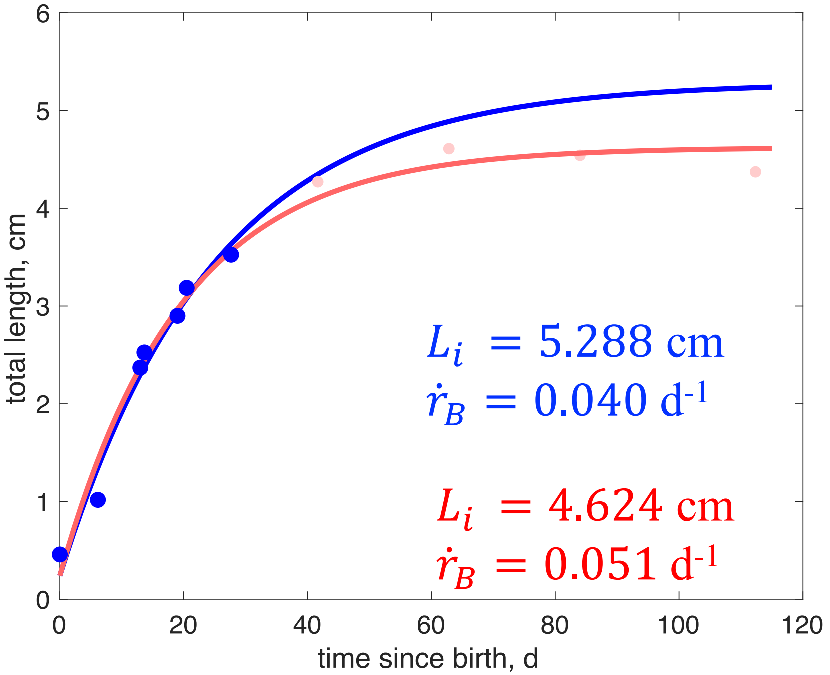

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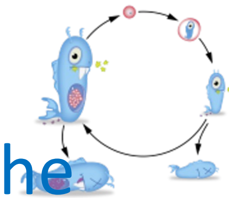

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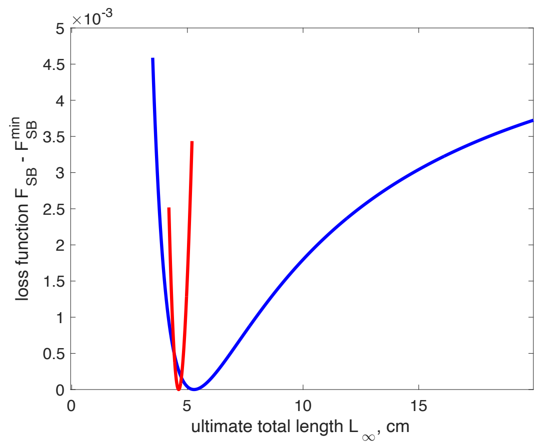

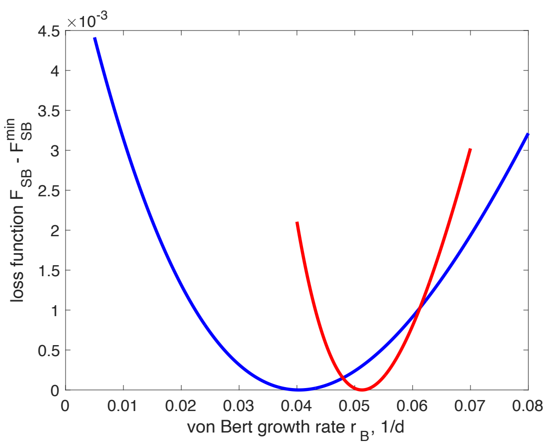

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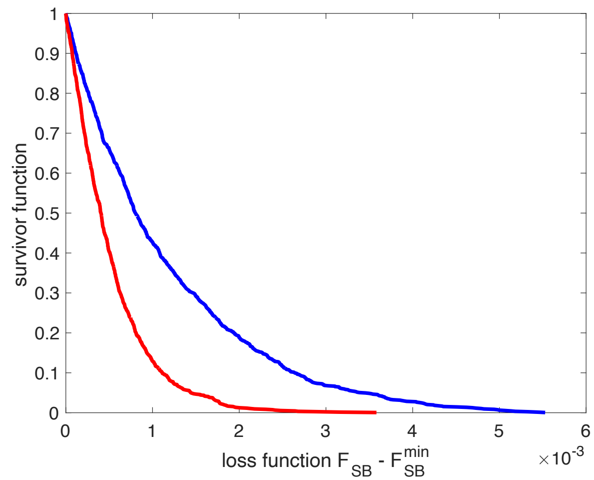
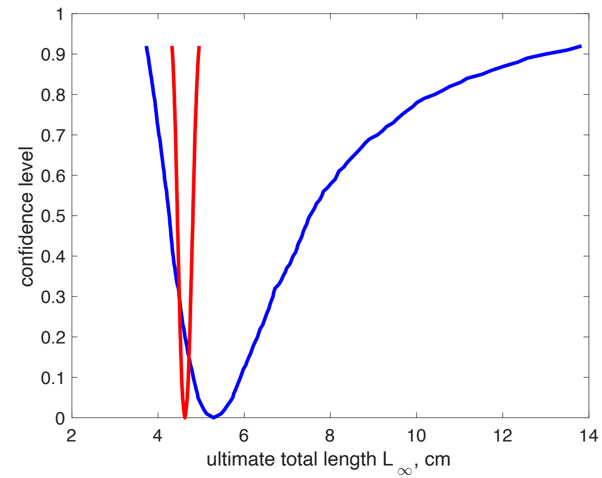
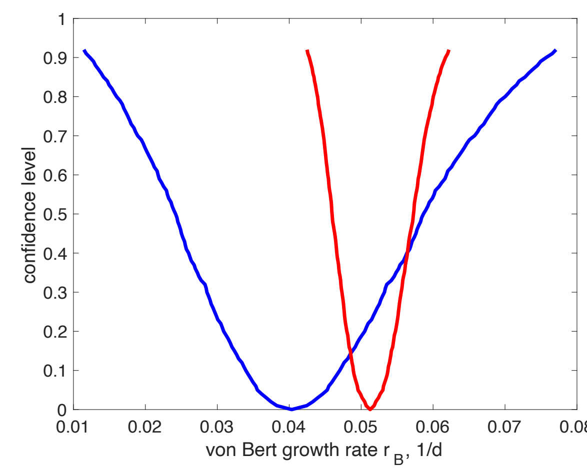




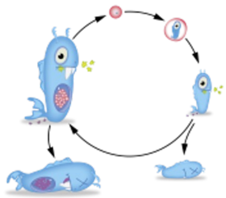


Full dataset

Reduced dataset

Example – reduced data set

sampled from the full curve



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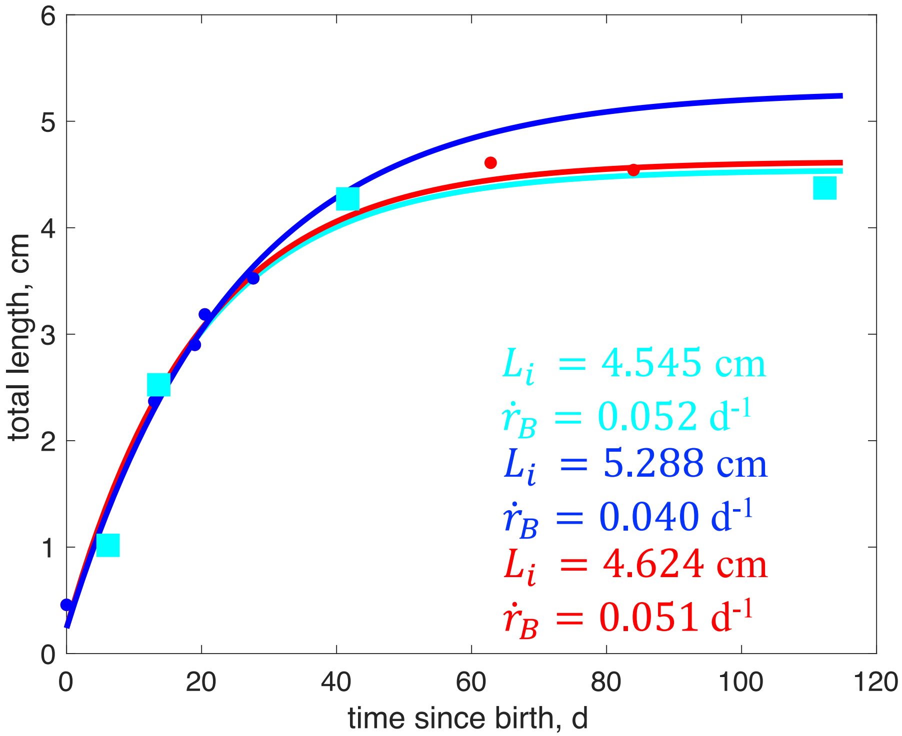

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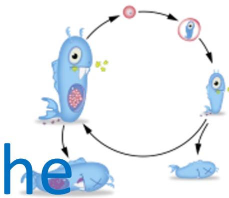

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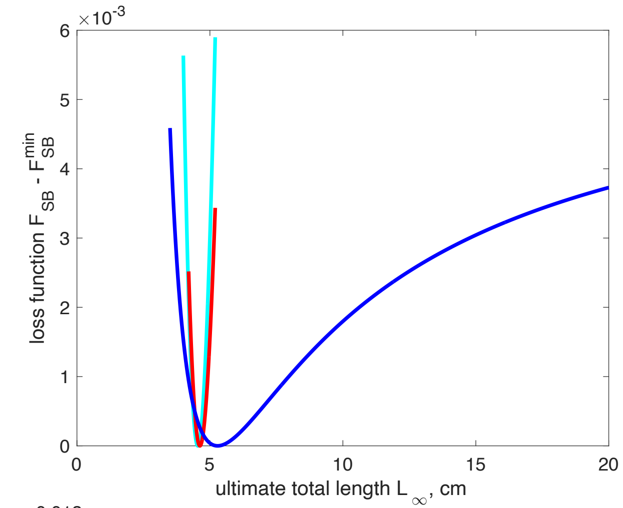

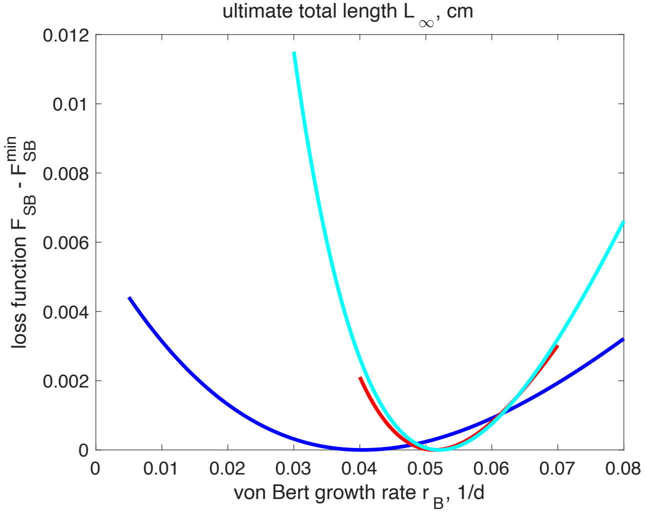

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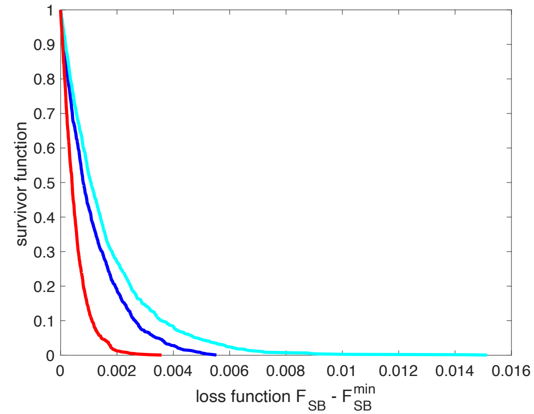

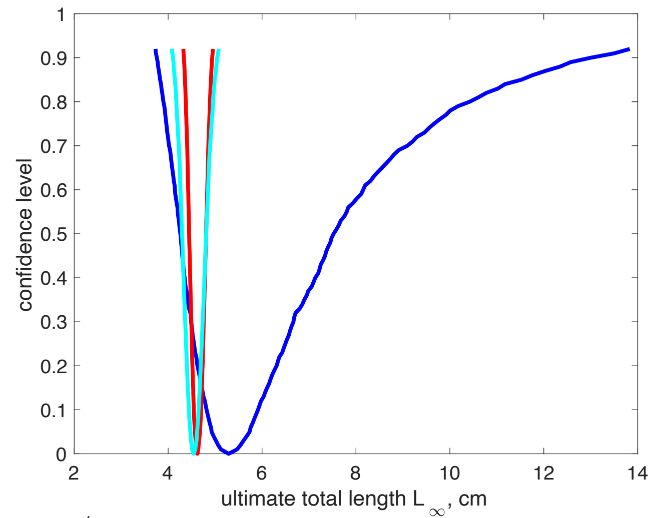

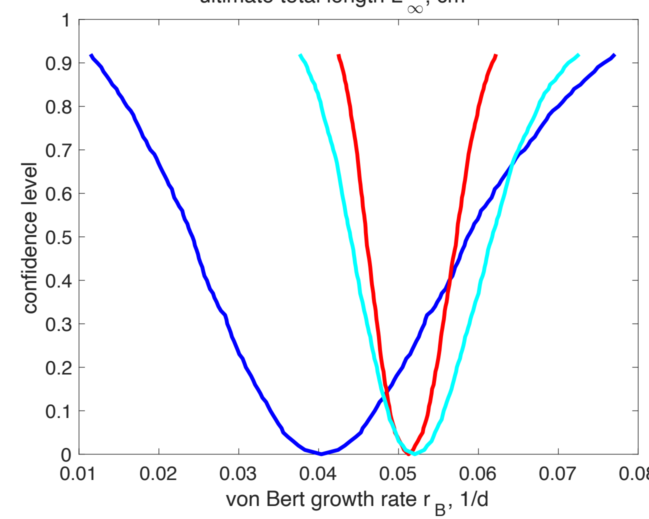

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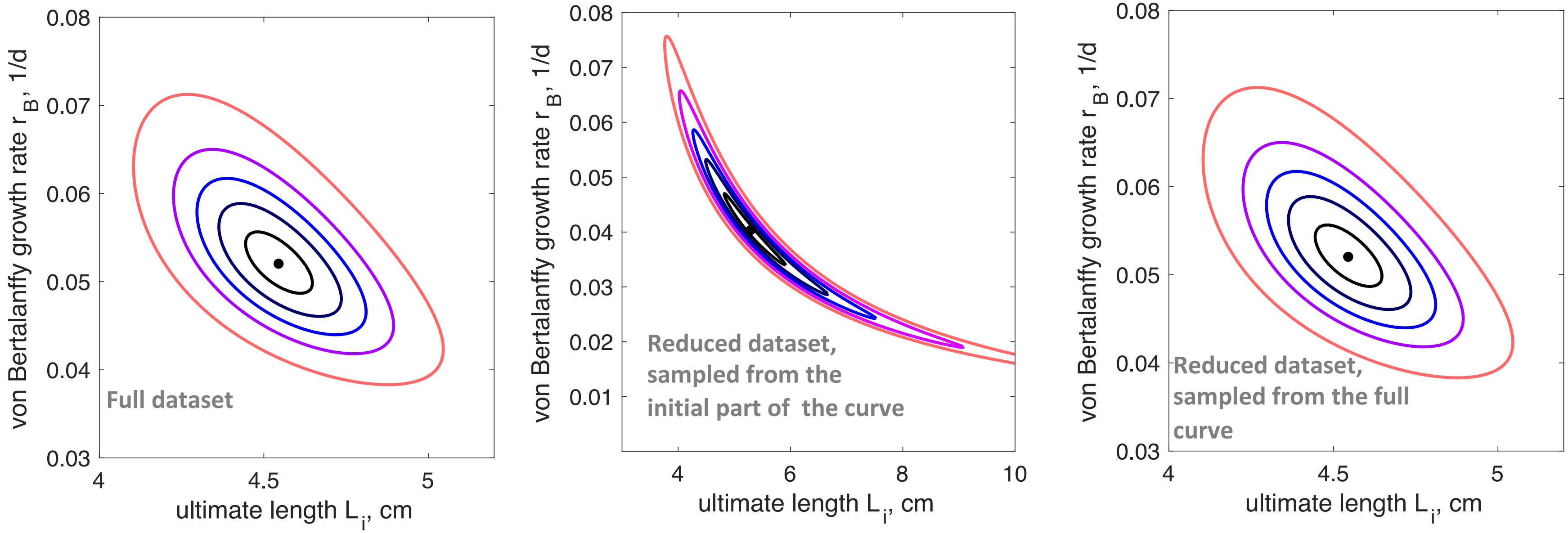
Reduced dataset,
sampled from the initial part of the curve

Reduced dataset,
sampled from the full curve

Confidence regions in 2D space

Contours of the loss function at various confidence leveles



Tool to compute profile-based CI

Your working folder must have:

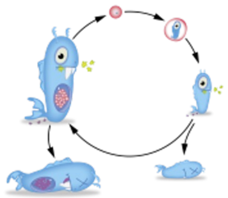
- the `run_CI.m` file located in **debtool_M/lib/pet_ci**

- and the 3 user-defined files of your species

  `mydata_my_pet.m`

  `predict_my_pet.m`

  `pars_init_my_pet.m` (with the best estimates) ξα σου



Thank you for your attention!!!

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